Head-on collisions of dark solitons near the zero-dispersion point in optical fibers

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The head-on collision of two *dark* solitons is studied in the normal-dispersion regime near the zero-dispersion point of a nonlinear single-mode optical fiber. Two different head-on collisions may actually occur. One is that of two dark solitons, and the other one is that of a dark with an antidark soliton. The solitons emerging from the collisions can preserve their original identities only to the second order. The phase shifts due to the collisions show different signs, depending on the ratio of the second- to third-order dispersion parameters of the system. [S1063-651X(96)10909-0]

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Although solitons arise in many areas of physics, single-mode optical fibers are known to be the most convenient quasi-one-dimensional systems to investigate dynamical properties of solitons, including their generation, evolution, and interaction. Moreover, potential applications, by using soliton pulses as information carriers in optical communication systems, have attracted considerable attention in recent years [1]. Hasegawa and Tappert [2] predicted that optical pulses may propagate in single-mode fibers without broadening in the form of bright and dark solitons, for which the nonlinear refractive index can compensate anomalous (negative) and normal (positive) group-velocity-dispersions, respectively. Bright solitons in optical fibers were first observed by Mollenauer, Stolen, and Gordon [3], and their interactions were studied both theoretically and experimentally [4–8]. Bright soliton propagation over a 10 000-km distance has been achieved by using an Er-doped fiber amplifier to compensate for fiber losses [9]. Dark solitons remained a mathematical curiosity until several years ago, when techniques for controlling both the phase and amplitude of subpicosecond pulses were applied in order to generate odd-symmetry dark pulses that propagated unmistakably as solitons [10]. Dark solitons have a number of interesting properties; e.g., they can be generated without a threshold condition in the pulse intensity similar to solitons of the Korteweg–de Vries (KdV) equation, they are less affected by losses or background fluctuations than bright solitons, and they are stable near the zero-dispersion (ZD) point (see, e.g., Kivshar [11]). Recently, the (2+1)-dimensional dark solitons, including dark-soliton strips and grids, optical vortex solitons, and ring dark solitons, have also been studied both theoretically and experimentally [12–14]. For optical communications it is important to understand the nature of the soliton interaction, because the interaction between closely spaced solitons may lead to pulse attraction and subsequent coalescence [15]. The interaction of dark solitons in the framework of the integrable nonlinear Schrödinger (NLS) equation was first considered by Blow and Doran [16] and Zhao and Bourkoff [17]. In Ref. [18], Thurston and Weiner reported numerical simulations that laid the groundwork for possible experiments aimed at the observation of dark-soliton collisions.

In optical fibers there are advantages to working near the ZD point, where the second-order dispersion is zero, because there the power required for creating bright solitons is significantly low. Since exact analytical solutions describing soliton propagation near the ZD point are not available, numerical and perturbative methods have been used. For dark solitons, Kivshar [19], and Kivshar and Afanasjev [20] analyzed the nonlinear dynamics of dark solitons near the ZD point by using a connection between the NLS equation and the KdV equation. They proved the existence of a new type of optical soliton, *antidark solitons*, i.e., dark solitons with the reverse sign in the amplitude. They pointed out that it is possible to observe a head-on collision between dark and antidark solitons in optical fibers in the normal dispersion regime near the ZD point. Numerically, they showed that such a head-on collision looks elastic for small-amplitude dark solitons.

In this paper, we *analytically* investigate the head-on collisions of dark solitons near the ZD point in optical fibers by using the Poincaré-Lighthill-Kuo (PLK) method [21,22]. We discuss the types of collisions and explicitly provide the postcollision phase shifts of each soliton together with the corrections of amplitude and phase functions of the optical pulses.

Using the slowly varying envelope approximation [1], the dimensionless envelope amplitude $u(x,t)$ of the electric field in the neighborhood of the ZD point in single-mode optical fibers satisfies the modified NLS equation

$$iu_x - \alpha u_{tt} + 2|u|^2 u - i\beta u_{ttt} = 0, \tag{1}$$

where the subscripts x and t represent partial derivatives. Time t in the reference frame moving with the group velocity is measured in units of the pulse duration T, and the coordinate x along a fiber is measured in units of $T/|k^{(1)}|$. The parameter $\alpha = k^{(2)}/(2T|k^{(1)}|)$ denotes the dimensionless second-order dispersion and $\beta = k^{(3)}/(6T^2|k^{(1)}|)$ represents the dimensionless third-order dispersion of the fiber. Here k is the propagation wave number, and $k^{(j)} = \partial^j k/\partial \omega^j$ $(j = 1,2,3)$.

When $\beta=0$ and $\alpha>0$, Eq. (1) is completely integrable and the one-soliton dark pulse takes the form

$$u(x,t)=u_0\,\frac{(\lambda-i\nu)^2+\exp Z}{1+\exp Z}\,\exp(2iu_0^2x), \tag{2}$$

where u_0 is the amplitude of constant-wave (CW) background, $\lambda=\sqrt{1-\nu^2}(0\leqslant\nu^2\leqslant1)$ and $Z=2\nu u_0(t-t_0-2\lambda\sqrt{\alpha}u_0x)/\sqrt{\alpha}$ (t_0 is an arbitrary constant). When $\nu=\pm1$, Eq. (2) describes the *fundamental* dark soliton $u=u_0\tanh[u_0(t-t_0)/\sqrt{\alpha}]\exp(2iu_0^2x)$. For $\nu^2\ll1$ it becomes the *gray* dark soliton

$$u=u_0[1+a_0(x,t)]\exp[2iu_0^2x+i\phi_0(x,t)], \tag{3}$$

$$a_0(x,t)=-\tfrac{1}{2}\nu^2\operatorname{sech}\tfrac{1}{2}Z, \tag{4}$$

$$\phi_0(x,t)=\nu[\tanh\tfrac{1}{2}Z-1], \tag{5}$$

$$Z=2\nu u_0[t-t_0\mp u_0\sqrt{\alpha}(2-\nu^2)x]/\sqrt{\alpha}. \tag{6}$$

Gray dark solitons can propagate with opposite directions, hence a head-on collision between two gray solitons is possible.

Using

$$u(x,t)=u_0[1+a(x,t)]\exp[2iu_0^2x+i\phi(x,t)], \tag{7}$$

Eq. (1) becomes

$$a_x-\alpha[(1+a)\phi_{tt}+2a_t\phi_t] -\beta[a_{ttt}-3a_t\phi_t^2-3(1+a)\phi_t\phi_{tt}]=0, \tag{8}$$

$$\phi_x-4u_0^2a+a\phi_x+\alpha[a_{tt}-(1+a)\phi_t^2]-2u_0^2(3a^2+a^3) -\beta[(1+a)\phi_{ttt}+3a_t\phi_{tt}+3a_{tt}\phi_t-(1+a)\phi_t^3]=0. \tag{9}$$

The system (8) and (9) can be reduced to the KdV equation under the weak nonlinearity and weak dispersion approximation [19]. Thus for the overtaking collision between the gray dark solitons in optical fibers, one can use the KdV equation derived in Ref. [19] and the inverse scattering transform [23] to obtain overtaking colliding effects of the solitons. However, for the head-on collision between the dark solitons, we must use some asymptotic expansion to solve the original equation of motion, Eq. (1), or equivalently, Eqs. (8) and (9).

We use the PLK method [21,22] to investigate the head-on collision between two dark solitons in optical fibers near the ZD point. Suppose that two dark solitons are far apart and heading toward each other. After some time they interact, following a collision, and then depart. Anticipating that the collision will result in phase-shifted trajectories [22] we set

$$\xi=\epsilon(t-c_Rx)+\epsilon^2P^{(0)}(\eta)+\epsilon^4P^{(1)}(\xi,\eta)+\cdots, \tag{10}$$

$$\eta=\epsilon(t+c_Lx)+\epsilon^2Q^{(0)}(\xi)+\epsilon^4Q^{(1)}(\xi,\eta)+\cdots, \tag{11}$$

where ϵ is the smallness and ordering parameter. The right- and left-running wave speeds $1/c_R$, $1/c_L$ are to be related to the amplitudes of the waves. The functions $P^{(j)}$ and $Q^{(j)}$ ($j=0,1,2,\ldots$) are to be determined in the process of our perturbational solution of Eqs. (8) and (9). These functions are introduced for the purpose of making uniformly valid asymptotic approximations (i.e., eliminating secular terms). One of the advantages of the PLK method over other ones is that it allows one to explicitly calculate postcollision effects such as phase shifts due to the head-on collision.

Using Eqs. (10) and (11) we obtain the transformation between derivatives as

$$\partial_x=\epsilon(-c_R\partial_\xi+c_L\partial_\eta)+\epsilon^3(c_LP_\eta^{(0)}\partial_\xi-c_RQ_\xi^{(0)}\partial_\eta)+\cdots, \tag{12}$$

$$\partial_t=\epsilon(\partial_\xi+\partial_\eta)+\epsilon^3(P_\eta^{(0)}\partial_\xi+Q_\xi^{(0)}\partial_\eta)+\cdots. \tag{13}$$

Introducing the asymptotic expansion

$$a=\epsilon^2(a^{(0)}+\epsilon^2a^{(1)}+\cdots),\quad \phi=\epsilon(\phi^{(0)}+\epsilon^2\phi^{(1)}+\cdots), \tag{14}$$

$$c_R=c+\epsilon^2R^{(1)}+\epsilon^4R^{(2)}+\cdots,$$
$$c_L=c+\epsilon^2L^{(1)}+\epsilon^4L^{(2)}+\cdots, \tag{15}$$

and substituting Eqs. (12)–(15) into Eqs. (8) and (9), one obtains a hierarchy of linear, inhomogeneous equations for $a^{(j)}$ and $\phi^{(j)}$ ($j=0,1,\ldots$). To leading order the solution is

$$a^{(0)}=f^{(0)}(\xi)+g^{(0)}(\eta), \tag{16}$$

$$\phi^{(0)}=-\frac{c}{\alpha}\int_{+\infty}^{\xi}f^{(0)}(\xi')d\xi'+\frac{c}{\alpha}\int_{-\infty}^{\eta}g^{(0)}(\eta')d\eta', \tag{17}$$

with $c=2u_0\sqrt{\alpha}$. The two functions $f^{(0)}$ and $g^{(0)}$ are to be determined. Thus to leading order, we have two waves, one of them, $f^{(0)}(\xi)$, is traveling to the right, and the other one, $g^{(0)}(\eta)$, is traveling to the left. The lower limits of integrations in Eq. (17) have been chosen to make the initial phases (before collision) of solitons $f^{(0)}(\xi)$ and $g^{(0)}(\eta)$ equal to zero.

In the next order, we have the equations for $a^{(1)}$ and $\phi^{(1)}$. The solvability conditions for $a^{(1)}$ and $\phi^{(1)}$ yield

$$-\tfrac{1}{3}\lambda_1^+Af_\xi^{(0)}+\lambda_1^+f^{(0)}f_\xi^{(0)}+\lambda_2^+f_{\xi\xi\xi}^{(0)}=0, \tag{18}$$

$$-\tfrac{1}{3}\lambda_1^-Bg_\eta^{(0)}+\lambda_1^-g^{(0)}g_\eta^{(0)}+\lambda_2^-g_{\eta\eta\eta}^{(0)}=0, \tag{19}$$

$$P_\eta^{(0)}=\left(\frac{1}{2}+\frac{3\beta c}{4\alpha^2}\right)g_\eta^{(0)}, \tag{20}$$

$$Q_\xi^{(0)}=\left(\frac{1}{2}-\frac{3\beta c}{4\alpha^2}\right)f_\xi^{(0)}, \tag{21}$$

with

$$R^{(1)}=c\left(1+\frac{\beta c}{2\alpha^2}\right)A,\quad L^{(1)}=c\left(1-\frac{\beta c}{2\alpha^2}\right)B, \tag{22}$$

and

$$\lambda_1^+ = 3c\left(1+\frac{\beta c}{2\alpha^2}\right), \quad \lambda_2^+ = -\left(\beta+\frac{\alpha^2}{2c}\right), \tag{23}$$

$$\lambda_1^- = -3c\left(1-\frac{\beta c}{2\alpha^2}\right), \quad \lambda_2^- = -\left(\beta-\frac{\alpha^2}{2c}\right), \tag{24}$$

where A and B are the constants related to the amplitudes of the solitons. Equations (18) and (19) are just the KdV equation in the reference frame of ξ and η, respectively. The soliton solutions are

$$f^{(0)}(\xi)=A\ \mathrm{sech}^2\left[\left(\frac{\lambda_1^+ A}{12\lambda_2^+}\right)^{1/2}\xi\right] \quad \text{(right-running soliton)}, \tag{25}$$

$$g^{(0)}(\eta)=B\ \mathrm{sech}^2\left[\left(\frac{\lambda_1^- B}{12\lambda_2^-}\right)^{1/2}\eta\right] \quad \text{(left-running soliton)}, \tag{26}$$

with $\mathrm{sgn}(A)=\mathrm{sgn}(\lambda_1^+\lambda_2^+)$ and $\mathrm{sgn}(B)=\mathrm{sgn}(\lambda_1^-\lambda_2^-)$. Then from Eqs. (20) and (21) we obtain

$$\begin{aligned} P^{(0)}(\eta) &= \left(\frac{1}{2}+\frac{3\beta c}{4\alpha^2}\right)\int_{-\infty}^{\eta} g^{(0)}(\eta')d\eta' \\ &= \left(\frac{1}{2}+\frac{3\beta c}{4\alpha^2}\right)\left(\frac{12\lambda_2^- B}{\lambda_1^-}\right)^{1/2} \\ &\quad \times\left\{\tanh\left[\left(\frac{\lambda_1^- B}{12\lambda_2^-}\right)^{1/2}\eta\right]+1\right\}, \end{aligned} \tag{27}$$

$$\begin{aligned} Q^{(0)}(\xi) &= \left(\frac{1}{2}-\frac{3\beta c}{4\alpha^2}\right)\int_{+\infty}^{\xi} f^{(0)}(\xi')d\xi' \\ &= \left(\frac{1}{2}-\frac{3\beta c}{4\alpha^2}\right)\left(\frac{12\lambda_2^+ A}{\lambda_1^+}\right)^{1/2} \\ &\quad \times\left\{\tanh\left[\left(\frac{\lambda_1^+ A}{12\lambda_2^+}\right)^{1/2}\xi\right]-1\right\}. \end{aligned} \tag{28}$$

From Eqs. (25)–(28), we can obtain the particular expressions of $a^{(1)}$ and $\phi^{(1)}$, but they are not needed here. Thus the solutions up to $O(\epsilon^4)$ order are

$$\begin{aligned} a(x,t) &= \epsilon^2[A\ \mathrm{sech}\,\theta_A(\xi)+B\ \mathrm{sech}\,\theta_B(\eta)] \\ &\quad +\epsilon^4\left\{\frac{\lambda_1^+}{3c}\left[1+\frac{\alpha}{\lambda_2^+ c}\left(\alpha+\frac{\beta c}{\alpha}\right)\right]A^2\mathrm{sech}^2\,\theta_A(\xi)\right. \\ &\quad -\frac{\lambda_1^-}{3c}\left[1-\frac{\alpha}{\lambda_2^- c}\left(\alpha-\frac{\beta c}{\alpha}\right)\right]B^2\mathrm{sech}^2\,\theta_B(\eta) \\ &\quad -\frac{1}{2}\left[3+\frac{\lambda_1^+\alpha}{\lambda_2^+ c^2}\left(\alpha+\frac{\beta c}{\alpha}\right)\right]A^2\mathrm{sech}^4\,\theta_A(\xi) \\ &\quad -\frac{1}{2}\left[3+\frac{\lambda_1^-\alpha}{\lambda_2^- c^2}\left(\alpha-\frac{\beta c}{\alpha}\right)\right]B^2\mathrm{sech}^4{}_B(\eta) \\ &\quad \left.+f^{(1)}(\xi)+g^{(1)}(\eta)\right\}+O(\epsilon^6), \end{aligned} \tag{29}$$

$$\begin{aligned} \phi(x,t) &= \epsilon\left\{-\frac{c}{\alpha}\left(\frac{12\lambda_2^+ A}{\lambda_1^+}\right)^{1/2}[\tanh\theta_A(\xi)-1]\right. \\ &\quad \left.+\frac{c}{\alpha}\left(\frac{12\lambda_2^- B}{\lambda_1^-}\right)^{1/2}[\tanh\theta_B(\eta)-1]\right\} \\ &\quad +\epsilon^3\left\{-\frac{c}{\alpha}\int_{+\infty}^{\xi} f^{(1)}(\xi')d\xi'\right. \\ &\quad \left.+\frac{c}{\alpha}\int_{-\infty}^{\eta} g^{(1)}(\eta')d\eta'\right\}+O(\epsilon^5), \end{aligned} \tag{30}$$

$$\begin{aligned} \xi &= \epsilon(t-c_R x)+\epsilon^2\left(\frac{1}{2}+\frac{3\beta c}{4\alpha^2}\right)\left(\frac{12\lambda_2^- B}{\lambda_1^-}\right)^{1/2}[\tanh\theta_B(\eta)+1] \\ &\quad +O(\epsilon^4), \end{aligned} \tag{31}$$

$$\begin{aligned} \eta &= \epsilon(t+c_L x)+\epsilon^2\left(\frac{1}{2}-\frac{3\beta c}{4\alpha^2}\right)\left(\frac{12\lambda_2^+ A}{\lambda_1^+}\right)^{1/2}[\tanh\theta_A(\xi)-1] \\ &\quad +O(\epsilon^4), \end{aligned} \tag{32}$$

$$c_R=c+\epsilon^2\tfrac{1}{3}\lambda_1^+ A+O(\epsilon^4), \quad c_L=c-\epsilon^2\tfrac{1}{3}\lambda_1^- B+O(\epsilon^4), \tag{33}$$

with

$$\theta_A(\xi)=\left(\frac{\lambda_1^+ A}{12\lambda_2^+}\right)^{1/2}\xi, \quad \theta_B(\eta)=\left(\frac{\lambda_1^- B}{12\lambda_2^-}\right)^{1/2}\eta, \tag{34}$$

where $f^{(1)}$ and $g^{(1)}$ are two functions to be determined in the next order.

The phase shifts following the head-on collision of two dark solitons can be obtained. Let us assume that the soliton $f^{(0)}$ (denoted by A) and soliton $g^{(0)}$ (denoted by B) are far enough from each other at the initial time ($t=-\infty$); i.e., soliton A is at $\xi=0$ and $\eta=-\infty$, and B is at $\eta=0$ and $\xi=+\infty$, respectively. After collision ($t=+\infty$), A is far to the right of B; i.e., A is at $\xi=0$, $\eta=+\infty$, and B is at $\eta=0$ and $\xi=-\infty$. Therefore, from Eqs. (31) and (32), we obtain the phase shifts of A and B, Δ_A and Δ_B:

$$\begin{aligned} \Delta_A &= \epsilon(t-c_R x)|_{\xi=0,\ \eta=+\infty}-\epsilon(t-c_R x)|_{\xi=0,\ \eta=-\infty} \\ &= -\epsilon^2\left(1+\frac{3\beta c}{2\alpha^2}\right)\left(\frac{12\lambda_2^- B}{\lambda_1^-}\right)^{1/2}, \end{aligned} \tag{35}$$

$$\begin{aligned} \Delta_B &= \epsilon(t+c_L x)|_{\eta=0,\ \xi=-\infty}-\epsilon(t+c_L x)|_{\eta=0,\ \xi=+\infty} \\ &= \epsilon^2\left(1-\frac{3\beta c}{2\alpha^2}\right)\left(\frac{12\lambda_2^+ A}{\lambda_1^+}\right)^{1/2}. \end{aligned} \tag{36}$$

As the crucial parameter is $\Gamma\equiv\alpha^{3/2}/(\beta u_0)$, which is the ratio of the second-order to third-order dispersion of the optical fiber, we come to the following conclusions:

(i) Soliton A, Eq. (25), is always a dark (*concave-up*) soliton traveling to the right. Soliton B, Eq. (26), traveling to the left, is dark when Γ is less than 1 or larger than 4. However, B becomes an antidark (*raised*) soliton if $1<\Gamma<4$.

(ii) When Γ is less than 1 or larger than 4, there is a head-on collision of two *dark* solitons A and B. When

$1<\Gamma<4$, we have a head-on collision between a *dark* soliton (A) and an *antidark* soliton (B).

(iii) The phase shift of soliton A due to the head-on collision is always negative. For soliton B, its phase shift is negative when $\Gamma<3$ but positive otherwise.

(iv) To the second order, head-on collisions are elastic but they become inelastic if higher-order corrections are included, in agreement with the numerical results reported by Kivshar and Afanasjev [20].

Needless to say, Γ is an important quantity to be controlled when experimentally studying the interaction of dark solitons in normal-dispersion regime near the ZD point in optical fibers. Noticing that in the standard single-mode silica-glass fiber, the ZD point is at wavelength $\lambda_{ZD}=1.27$ μm. Near λ_{ZD}, $k^{(2)}\approx 9\times[1.27-\lambda_0(\mu\text{m})]\times 10^{-26}$ (sec^2/m), $k^{(3)}\approx 2.3\times\sqrt{\lambda_0(\mu\text{m})}[\lambda_0(\mu\text{m})-1]\times 10^{-40}$ (sec$^3/m$), n_2(Kerr coefficient)$=1.2\times 10^{-22}$ (m/V)^2 [24]. Dimensional electric field in Eq. (1) is defined by $E=2(|k^{(1)}|c/T\omega_0 n_2)^{1/2}u$, where c is the speed of light in vacuum and $\omega_0=2\pi c/\lambda_0$ (λ_0 is the wavelength of carrier wave). From the preceding results we can see that the experimental observation of the head-on collision between dark solitons near the ZD point will require a peak electric field E_0 around $(7/\Gamma)\times 10^7$ (V/m) if we take $\lambda_{ZD}-\lambda_0=0.01$ μm and $E_0\approx(2/\Gamma)\times 10^6$ (V/m) when $\lambda_{ZD}-\lambda_0=0.001$ μm. On the other hand, as suggested in Ref. [18], in order to achieve an observable phase shift in experiment, the blackness parameter of the dark pulse, controlled by ϵ^2 [if A and B are fixed, see Eq. (29)] in our case, cannot be too small.

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